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CERTAIN FINITE INTEGRAL FORMULAS PERTAINING TO THE PRODUCT OF A GENERALIZED BESSEL-MAITLAND FUNCTION AND JACOBI POLYNOMIAL

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Abstract: The main object of this paper is to evaluate certain finite single and double integral formulas involving the product of a generalized Bessel-Maitland function and the classical Jacobi polynomial. The outcomes of proposed integrals are expressed in terms of the well-known Srivastava and Daoust function. Several interesting special integrals are obtained as the particular cases of the results established in the present investigation.

Keywords and Phrases: Generalized Bessel-Maitland function, Jacobi polynomial, Srivastava and Daoust function.

2020 Mathematics Subject Classification: 33C10, 33C45, 33C65.

1. Introduction

Special functions are ubiquitous in the applied sciences. A number of special functions are compiled and presented in classical monographs [6, 14, 38]. Being the fundamental components of applicable mathematics, integrals and derivatives of special functions play a key role in diversified fields of science and technology. Due to a wide variety of applications, the Bessel function and its numerous generalizations are studied by distinguished researchers. For instance, Suthar et al. [33] have proposed an extension of the Bessel-Maitland function and investigated the associated fractional calculus. Further, Tilahun et al. [36] proposed another extension of a generalized form of the Bessel-Maitland function and established several interesting results for the connected fractional integral and differential operators. Also, Suthar et al. [35] have discussed Pathway fractional integral formulae involving extended Bessel-Maitland function in the kernel. Moreover, Marimuthu et al. [16] have generalized a new family of meromorphic multivalent functions defined by Bessel function (see also [37]). For some other related extensions of the Bessel function, we refer [4, 32, 34].

A variety of integrals involving generalized special functions emerges during the solutions of distinct mathematical problems pertaining to numerous fields of applied sciences, including physical sciences, finance, social sciences, computational biology, and physical chemistry. For instance, Elliptic-type integrals have their potential application in various radiation field-related problems [3, 31] (see also, [9]). A remarkably large number of integral formulas involving a variety of special functions have been developed by several researchers. For some more integrals involving the product of different special functions and polynomials, we refer [7, 11, 12, 20].

Motivated by the work of other co-researchers (see, for e.g., [2, 18]), in the present research work, we have proposed two theorems based on single and double finite integrals involving the product of a generalized Bessel-Maitland function and the classical Jacobi polynomial with a suitably generalized argument. Some of the interesting special cases are also demonstrated in terms of corollaries of the theorems derived in the current investigation.
The generalized Bessel-Maitland function $J^{\nu}_{\omega}(z)$ is defined as [17]:

$$J^{\nu}_{\omega}(z)=\sum_{n=0}^{\infty}\frac{(-z)^n}{n!\Gamma(\omega n+\nu+1)},\qquad (z\in\mathbb{C},\omega>0). \tag{1.1}$$

A generalization of the Bessel-Maitland function defined by Pathak [22] (for more details, see [33], [34]) is

$$J^{\nu,\gamma}_{\omega,q}(z)=\sum_{n=0}^{\infty}\frac{(-z)^n(\gamma)_{qn}}{n!\Gamma(\omega n+\nu+1)}, \tag{1.2}$$

with $\omega,\nu,\gamma\in\mathbb{C},\ \ Re(\gamma)>0,\ Re(\omega)>0,\ Re(\nu)\geq -1$, and $q\in(0,1)\cup\mathbb{N}$.
Also, $(\gamma)_{qn}$ involved in (1.2) is the generalized Pochhammer symbol (see [30], p.6, eq.30) and given as:

$$(\gamma)_{qn}=\frac{\Gamma(\gamma+qn)}{\Gamma(\gamma)}=q^{qn}\prod_{j=1}^{q}\left(\frac{\gamma+j-1}{q}\right)_n,\qquad n\geq 0,\ q\in\mathbb{N}. \tag{1.3}$$

Particularly, the classical Pochhammer symbol $(\gamma)_n$ is given as [24, 30]:

$$(\gamma)_n = \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)} = \begin{cases} 1 & \text{if } \ n=0, \\ \gamma(\gamma+1)...(\gamma+n-1) & \text{if } \ n\in\mathbb{N}. \end{cases} \tag{1.4}$$

Ghayasuddin and Khan [8] investigated the following extension of Bessel-Maitland function:

$$J^{\nu,\gamma,\delta}_{\omega,q,p}(z) = \sum_{n=0}^{\infty} \frac{(-z)^n(\gamma)_{qn}}{\Gamma(\omega n+\nu+1)(\delta)_{pn}}, \tag{1.5}$$

where $\omega,\nu,\gamma,\delta\in\mathbb{C}$, $Re(\gamma)>0$, $Re(\delta)>0$, $Re(\omega)>0$, $Re(\nu)\geq -1$, $p,q>0$, and $q<Re(\omega)+p$.

In a recent investigation, Ali et al. [1] have investigated the following generalized form of the Bessel-Maitland function:

$$J^{\nu,\gamma,\delta,\rho}_{\omega,q,p,\xi}(z) = \sum_{n=0}^{\infty} \frac{(-z)^n(\gamma)_{qn}(\rho)_{\xi n}}{\Gamma(\omega n+\nu+1)(\delta)_{pn}}, \tag{1.6}$$

where $\omega,\nu,\gamma,\delta,\rho\in\mathbb{C}$, $Re(\gamma)>0$, $Re(\delta)>0$, $Re(\rho)>0$, $Re(\omega)>0$, $Re(\nu)\geq -1$, $\xi,p,q\geq 0$, and $\xi,p<Re(\omega)+q$.

The generalized Bessel-Maitland function given in (1.6) is closely connected with the different forms of generalized Mittag-Leffler functions. Now, we discuss some of the interesting particular relationships of the generalized Bessel-Maitland function with the other well-known Mittag-Leffler functions.

For $\rho=\xi=1$, and replacing ν by $\nu-1$ in (1.6), it reduces in the following generalized Mittag-Leffler function [25]:

$$J^{\nu-1,\gamma,\delta,1}_{\omega,q,p,1}(-z) = E^{\nu,\gamma,\delta}_{\omega,q,p}(z) = \sum_{n=0}^{\infty} \frac{(z)^n(\gamma)_{qn}}{\Gamma(\omega n+\nu)(\delta)_{pn}}. \tag{1.7}$$

On putting $\rho=\xi=1$, $q=p=1$, and replacing ν by $\nu-1$ in (1.6), we arrive at the following relationship:

$$J^{\nu-1,\gamma,\delta,1}_{\omega,1,1,1}(-z) = E^{\gamma,\delta}_{\nu,\omega}(z) = \sum_{n=0}^{\infty} \frac{(z)^n(\gamma)_{n}}{\Gamma(\omega n+\nu)(\delta)_{n}}, \tag{1.8}$$

where $E^{\gamma,\delta}_{\nu,\omega}(z)$ is the four-parameter Mittag-Leffler function defined by Salim [26].

On setting $\rho=\xi=1$, $\delta=p=1$, and replacing ν by $\nu-1$ in (1.6), we get the following relationship:

$$J^{\nu-1,\gamma,1,1}_{\omega,q,1,1}(-z) = E^{\gamma,q}_{\nu,\omega}(z) = \sum_{n=0}^{\infty} \frac{(z)^n(\gamma)_{qn}}{\Gamma(\omega n+\nu)}, \tag{1.9}$$

where $E_{\nu,\omega}^{\gamma,q}(z)$ is the four-parameter Mittag-Leffler function investigated by Shukla and Prajapati [27].
On taking $\rho = \xi = 1, q = 1, \delta = p = 1$, and replacing ν by $\nu - 1$ in (1.6), we obtain the following relationship:

$$J_{\omega,1,1,1}^{\nu-1,\gamma,1,1}(-z) = E_{\nu,\omega}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(z)^n (\gamma)_n}{\Gamma(\omega n + \nu)}, \tag{1.10}$$

where $E_{\nu,\omega}^{\gamma}(z)$ is the three-parameter Mittag-Leffler function introduced by Prabhakar [23].
Also, on setting $\gamma = q = 1, \rho = \xi = 1, \delta = p = 1$, and replacing ν by $\nu - 1$ in equation (1.6), we get the following relationship:

$$J_{\omega,1,1,1}^{\nu-1,1,1,1}(-z) = E_{\nu,\omega}(z) = \sum_{n=0}^{\infty} \frac{(z)^n}{\Gamma(\omega n + \nu)}, \tag{1.11}$$

where $E_{\nu,\omega}(z)$ is the two-parameter Mittag-Leffler function, defined by Wiman [39].
Now, on putting $\nu = 0,\ \omega = 1$, and $\rho = \xi = 1$ in (1.6), we get the following relationship between the generalized Bessel-Maitland function and the Mittag-Leffler confluent hypergeometric function [10]:

$$J_{1,q,p,1}^{0,\gamma,\delta,1}(-z) = E_{q,p}^{\gamma,\delta}(z) = \sum_{n=0}^{\infty} \frac{(z)^n (\gamma)_{qn}}{n!(\delta)_{pn}}, \tag{1.12}$$

where $\gamma, \delta, q, p \in \mathbb{C}$.
The Jacobi polynomial $P_n^{(\alpha,\beta)}(z)$ of degree n and order $(\alpha > -1), (\beta > -1)$ is defined as [24]:

$$P_n^{(\alpha,\beta)}(z) = \sum_{k=0}^{n} \frac{(1+\alpha)_n (1+\alpha+\beta)_{n+k}}{k!(n-k)!(1+\alpha)_k (1+\alpha+\beta)_n} \left(\frac{z-1}{2}\right)^k. \tag{1.13}$$

It can be shown that for certain values of the parameters α and β in (1.13), the Jacobi polynomial reduces into several interesting polynomials which include the Ultraspherical polynomial, Gegenbauer polynomial, Chebyshev polynomial of first and second kind, and Legendre polynomial etc. (for more details, see [24]).

Srivastava and Daoust [28] proposed a multivariate generalization of the hypergeometric function, known as the Srivastava and Daoust function in the literature, is defined as:

$$F_{C:D^1;\ldots;D^r}^{A:B^1;\ldots;B^r} \begin{bmatrix} (a:\theta^1,\ldots,\theta^r):(b^1:\phi^1);\ldots;(b^r:\phi^r); & \\ & x_1,\ldots,x_r \\ (c:\varphi^1,\ldots,\varphi^r):(d^1:\delta^1);\ldots;(d^r:\delta^r); & \end{bmatrix}$$

$$= \sum_{k_1,...,k_r=0}^{\infty} \frac{\Pi_{j=1}^{A}(a_j)_{k_1\theta_j^1+...+k_r\theta_j^{(r)}} \Pi_{j=1}^{B^1}(b_j^1)_{k_1\phi_j^1} ... \Pi_{j=1}^{B^r}(b_j^{(r)})_{k_r\phi_j^{(r)}}}{\Pi_{j=1}^{C}(c_j)_{k_1\varphi_j^1+...+k_r\varphi_j^{(r)}} \Pi_{j=1}^{D^1}(d_j^1)_{k_1\delta_j^1} ... \Pi_{j=1}^{D^r}(d_j^{(r)})_{k_r\delta_j^{(r)}}} \frac{x_1^{k_1}}{k_1!} \cdots \frac{x_r^{k_r}}{k_r!}. \quad (1.14)$$

In the above series form of the Srivastava and Daoust function the coefficients $\theta_j^{(l)}$, $j = 1, ..., A$; $\phi_j^{(l)}$, $j = 1, ..., B^{(l)}$; $\varphi_j^{(l)}$, $j = 1, ..., C$; $\delta_j^{(l)}$, $j = 1, ..., D^{(l)}$ are real and positive, and (a) abbreviates the array of A parameters $a_1, ..., a_A$, (b^l) abbreviates the array of $B^{(l)}$ parameters $b_j^{(l)}$, $j = 1, ..., B^{(l)}$; $\forall\, l \in \{1, ..., r\}$, with similar interpretations for (c) and $(d^{(l)})$, $\forall\, l \in \{1, ..., r\}$; etc. For applications of Srivastava and Daoust function, we refer Srivastava and Daoust [29], Khan et al. [11], Pandey [19] and Chaurasia and Pandey [3].

Let us recall the following two lemmas, recently obtained in [21] as the extensions of classical integrals proposed by MacRobert [15] and Edward [5], respectively.

Lemma 1.1. *If $Re(\zeta) > 0$, $Re(\eta) > 0$, $a_1 \neq a_2$, $\mu^* \in \mathbb{R}\backslash \{-\mu, -\lambda\}$, and the constant λ and μ are such that the values of the parameters λ, μ and μ^* cannot all be zero simultaneously, (where $a_1 \leq t \leq a_2$) then the following result holds:*

$$\int_{a_1}^{a_2} (t - a_1)^{\zeta-1}(a_2 - t)^{\eta-1}[\mu^*(a_2 - a_1) + \lambda(t - a_1) + \mu(a_2 - t)]^{-\zeta-\eta} dt$$

$$= \frac{\Gamma(\zeta)\Gamma(\eta)}{(a_2 - a_1)(\mu^* + \lambda)^{\zeta}(\mu^* + \mu)^{\eta}\Gamma(\zeta + \eta)}. \quad (1.15)$$

Lemma 1.2. *If $Re(\zeta) > 0$, $Re(\eta) > 0$, and $a_1 \neq a_2$, where $a_1 \leq u \leq a_2$, $a_1 \leq v \leq a_2$, then there holds the following integral formula:*

$$\int_{a_1}^{a_2} \int_{a_1}^{a_2} (u - a_1)^{\zeta}(a_2 - v)^{\zeta-1}(a_2 - u)^{\eta-1} \left[1 - \frac{(u - a_1)(v - a_1)}{(a_2 - a_1)^2}\right]^{1-\zeta-\eta} dudv$$

$$= (a_2 - a_1)^{2\zeta+\eta} \frac{\Gamma(\zeta)\Gamma(\eta)}{\Gamma(\zeta + \eta)}. \quad (1.16)$$

2. Main Result

In this section, we have established two theorems based on the lemmas 1.1 and 1.2, respectively. Proposed theorems involve a single and a double finite integral pertaining to the product of a generalized Bessel-Maitland function (1.6) and Jacobi polynomial (1.13) with suitably generalized arguments. The outcomes are presented in terms of the well-known Srivastava and Daoust hypergeometric function.

Theorem 2.1. *For $Re(\eta) > 0$, $Re(\zeta) > 0$, $Re(\gamma) > 0, Re(\delta) > 0, Re(\rho) > 0,$*

$Re(\omega) > 0$, $Re(\nu) \geq -1$, $\omega, \nu, \gamma, \delta, \rho \in \mathbb{C}$, $\alpha, \beta, \xi, p, q \geq 0$, $\mu^* \in \mathbb{R}\backslash\{-\mu, -\lambda\}$, $\lambda_1, \lambda_2 > 0$, $a_1 \neq a_2$, and the constant λ and μ are such that the values of the parameters λ, μ and μ^* cannot all be zero simultaneously, then for $a_1 \leq t \leq a_2$ there holds the following integral formula:

$$\int_{a_1}^{a_2} (t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times J_{\omega,q,p,\xi}^{\nu,\gamma,\delta,\rho}\left[\frac{(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]dt$$

$$= \frac{(\mu^*+\lambda)^{(-\zeta)}(\mu^*+\mu)^{(-\eta)}}{(a_2-a_1)\Gamma(\nu+1)}B(\zeta,\eta)$$

$$\times F_{1+\omega+p+(\lambda_1+\lambda_2):0;1}^{2+\lambda_1+\lambda_2+\xi+q:0;0}\Bigg[\begin{matrix}[\Delta(\lambda_1;\zeta):1,2],[\Delta(\lambda_2;\eta):1,2],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta):1,2],\end{matrix}$$

$$[\Delta(\xi;\rho):1,1],[1+\alpha+\beta:1,2],[\Delta(q,\gamma):1,1],$$

$$[\Delta(\omega;\nu+1):1,1],[\Delta(p;\delta):1,1],[1+\alpha+\beta:1,1]$$

$$\begin{matrix}[1+\alpha:1,1]:\_;\_;\\ :\_;[1+\alpha;1];\end{matrix}\quad \frac{-(\lambda_1{}^{\lambda_1})(\lambda_2{}^{\lambda_2})q^q\xi^\xi}{p^p\omega^\omega(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(\mu^*+\lambda)^{\lambda_1}(\mu^*+\mu)^{\lambda_2}},$$

$$\left.\frac{(\lambda_1{}^{2\lambda_1})(\lambda_2{}^{2\lambda_2})q^q\xi^\xi}{p^p\omega^\omega(\mu^*+\lambda)^{2\lambda_1}(\mu^*+\mu)^{2\lambda_2}(\lambda_1+\lambda_2)^{2(\lambda_1+\lambda_2)}}\right], \tag{2.1}$$

where $J_{\omega,q,p,\xi}^{\nu,\gamma,\delta,\rho}(\cdot)$ and $P_n^{(\alpha,\beta)}(\cdot)$ involved in LHS of (2.1) are the generalized Bessel-Maitland function and Jacobi polynomial defined in (1.6) and (1.13), respectively. Also the function $F_{1+(\lambda_1+\lambda_2)+\omega+p:0;1}^{2+\lambda_1+\lambda_2+\xi+q:0;0}(\cdot)$ involved in RHS of (2.1) is the well-known Srivastava and Daoust function defined in (1.14). Further, the symbol $\Delta(p;\tau)$ is a p-tuple $\frac{\tau}{p}, \frac{\tau+1}{p}, ..., \frac{\tau+p-1}{p}$ with $p \geq 1$ and $\Delta(q;\nu+1)$ is a q-tuple $\frac{\nu+1}{q}, \frac{\nu+2}{q}, ..., \frac{\nu+q}{q}$ with $q \geq 1$.

Proof. To prove Theorem 2.1, we first express the generalized Bessel-Maitland function and the Jacobi polynomial in the series form given by (1.6) and (1.13), respectively. Now, on interchanging the order of integration and summation, which is valid under the given conditions, we may write

$$\int_{a_1}^{a_2}(t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times\sum_{n=0}^{\infty}\frac{(-1)^n(\gamma)_{qn}(\rho)_{\xi n}(t-a_1)^{n\lambda_1}(a_2-t)^{n\lambda_2}}{\Gamma(\omega n+\nu+1)(\delta)_{pn}[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{n(\lambda_1+\lambda_2)}}$$

$$\times\sum_{k=0}^{n}\frac{(-1)^k(1+\alpha)_n(1+\alpha+\beta)_{n+k}}{k!(n-k)!(1+\alpha)_k[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{k(\lambda_1+\lambda_2)}}$$

$$\times\frac{(t-a_1)^{k\lambda_1}(a_2-t)^{k\lambda_2}}{(1+\alpha+\beta)_n}dt=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(-1)^{n+k}(1+\alpha)_n}{k!(n-k)!(1+\alpha)_k(1+\alpha+\beta)_n}$$

$$\times\frac{(1+\alpha+\beta)_{n+k}(\gamma)_{qn}(\rho)_{\xi n}}{\Gamma(\omega n+\nu+1)(\delta)_{pn}}\int_{a_1}^{a_2}(t-a_1)^{\zeta+n\lambda_1+k\lambda_1-1}(a_2-t)^{\eta+n\lambda_2+k\lambda_2-1}$$

$$[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta-n(\lambda_1+\lambda_2)-k(\lambda_1+\lambda_2)}dt. \tag{2.2}$$

Now, we evaluate the integration involved in RHS of (2.2), using Lemma 1.1, we may rewrite RHS of (2.2) as the following expression:

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(-1)^{n+k}(1+\alpha)_n(1+\alpha+\beta)_{n+k}(\gamma)_{qn}(\rho)_{\xi n}}{k!(n-k)!(1+\alpha)_k(1+\alpha+\beta)_n\Gamma(\omega n+\nu+1)(\mu^*+\lambda)^{(\zeta+n\lambda_1+k\lambda_1)}}$$

$$\times\frac{\Gamma(\zeta+n\lambda_1+k\lambda_1)\Gamma(\eta+n\lambda_2+k\lambda_2)}{(\delta)_{pn}(a_2-a_1)(\mu^*+\mu)^{(\eta+n\lambda_2+k\lambda_2)}\Gamma\left(\zeta+\eta+n(\lambda_1+\lambda_2)+k(\lambda_1+\lambda_2)\right)}. \tag{2.3}$$

Moreover, using the following well-known identity [24] in RHS of (2.3):

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}B(k,n)=\sum_{n,k=0}^{\infty}B(k,n+k), \tag{2.4}$$

we can express RHS of (2.3) as:

$$=\frac{\Gamma(\zeta)\Gamma(\eta)}{(a_2-a_1)(\mu^*+\lambda)^{(\zeta)}(\mu^*+\mu)^{(\eta)}\Gamma(\zeta+\eta)}$$

$$\sum_{n,k=0}^{\infty}\frac{(-1)^{n+2k}(1+\alpha)_{n+k}(1+\alpha+\beta)_{n+2k}}{k!n!\Gamma(\omega(n+k)+\nu+1)(1+\alpha)_k(1+\alpha+\beta)_{n+k}}$$

$$\times \frac{(\zeta)_{\lambda_1(n+2k)}(\eta)_{\lambda_2(n+2k)}(\gamma)_{q(n+k)}(\rho)_{\xi(n+k)}}{(\delta)_{p(n+k)}(\mu^*+\lambda)^{(n\lambda_1+2k\lambda_1)}(\mu^*+\mu)^{(n\lambda_2+2k\lambda_2)}(\zeta+\eta)_{(\lambda_1+\lambda_2)(n+2k)}} \tag{2.5}$$

Finally, RHS of (2.5) can be represented in terms of the Srivastava and Daoust function with the help of (1.14), we arrive at the desired form presented as RHS of (2.1). This completes the proof of Theorem 2.1.

Theorem 2.2. *For $Re(\zeta) > 0$, $Re(\eta) > 0$, $Re(\gamma) > 0$, $Re(\delta) > 0, Re(\rho) > 0$, $Re(\omega) > 0$, $Re(\nu) \geq -1$, $\omega, \nu, \gamma, \delta, \rho \in \mathbb{C}$, $\alpha, \beta, \xi, p, q \geq 0$, $\lambda_1, \lambda_2 > 0$, and $a_1 \neq a_2$, then for $a_1 \leq u \leq a_2, a_1 \leq v \leq a_2$, there holds the following integral formula:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times J^{\nu,\gamma,\delta,\rho}_{\omega,q,p,\xi}\left[\frac{(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P^{(\alpha,\beta)}_{n}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right] dudv$$

$$= \frac{(a_2-a_1)^{(2\zeta+\eta)}}{\Gamma(\nu+1)}B(\zeta,\eta)F^{2+\lambda_1+\lambda_2+\xi+q:0;0}_{1+(\lambda_1+\lambda_2)+\omega+p:0;1}\left[\begin{matrix}[\Delta(\lambda_1;\zeta):1,2],[\Delta(\lambda_2;\eta):1,2], \\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta):1,2],\end{matrix}\right.$$

$$[\Delta(\xi;\rho):1,1],[\Delta(q,\gamma):1{,}1],[1+\alpha+\beta:1,2],[1+\alpha:1,1]:\_;\_;$$

$$[\Delta(p;\delta):1,1],[\Delta(\omega;\nu+1):1,1],[1+\alpha+\beta:1,1]:\_;[1+\alpha;1];$$

$$\left.\frac{-(\lambda_1{}^{\lambda_1})(\lambda_2{}^{\lambda_2})q^q\xi^\xi}{p^p\omega^\omega(a_2-a_1)^{-(2\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}},\frac{(\lambda_1{}^{2\lambda_1})(\lambda_2{}^{2\lambda_2})q^q\xi^\xi}{p^p\omega^\omega(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{2(\lambda_1+\lambda_2)}}\right] \tag{2.6}$$

Proof. To prove Theorem 2.2, we first express the generalized Bessel-Maitland function and Jacobi polynomial in series form, given in (1.6) and (1.13), respectively. Now, on interchanging the order of integration and summation, which is valid under the given conditions, we get

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times \sum_{n=0}^{\infty} \frac{(-1)^n (\gamma)_{qn} (\rho)_{\xi n} (u-a_1)^{n\lambda_1} (a_2-v)^{n\lambda_1} (a_2-u)^{n\lambda_2}}{\Gamma(\omega n+\nu+1)(\delta)_{pn} \left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{n(\lambda_1+\lambda_2)}}$$

$$\times \sum_{k=0}^{n} \frac{(-1)^k (1+\alpha)_n (1+\alpha+\beta)_{n+k} (u-a_1)^{k\lambda_1} (a_2-v)^{k\lambda_1} (a_2-u)^{k\lambda_2}}{k!(n-k)!(1+\alpha)_k (1+\alpha+\beta)_n \left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{k(\lambda_1+\lambda_2)}} dudv$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(-1)^{n+k} (1+\alpha)_n (1+\alpha+\beta)_{n+k} (\gamma)_{qn} (\rho)_{\xi n}}{k!(n-k)!(1+\alpha)_k (1+\alpha+\beta)_n (\delta)_{pn} \Gamma(\omega n+\nu+1)}$$

$$\int_{a_1}^{a_2} \int_{a_1}^{a_2} (u-a_1)^{\zeta+n\lambda_1+k\lambda_1} (a_2-v)^{\zeta+n\lambda_1+k\lambda_1-1} (a_2-u)^{\eta+n\lambda_2+k\lambda_2-1}$$

$$\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta-n(\lambda_1+\lambda_2)-k(\lambda_1+\lambda_2)} dudv. \tag{2.7}$$

Using Lemma 1.2 and the identity (2.4) in RHS of the above (2.7), we get the following expression as RHS of (2.7):

$$= \frac{\Gamma(\zeta)\Gamma(\eta)(a_2-a_1)^{(2\zeta+\eta)}}{\Gamma(\zeta+\eta)} \sum_{n,k=0}^{\infty} \frac{(-1)^{n+2k} (1+\alpha)_{n+k} (1+\alpha+\beta)_{n+2k}}{k!n!(1+\alpha)_k (1+\alpha+\beta)_{n+k}}$$

$$\times \frac{(a_2-a_1)^{(n(2\lambda_1+\lambda_2)+2k(2\lambda_1+\lambda_2))} (\gamma)_{q(n+k)} (\rho)_{\xi(n+k)} (\zeta)_{\lambda_1(n+2k)} (\eta)_{\lambda_2(n+2k)}}{\Gamma(\omega(n+k)+\nu+1)(\delta)_{p(n+k)} (\zeta+\eta)_{(n(\lambda_1+\lambda_2)+2k(\lambda_1+\lambda_2))}}.$$

By applying the definition of the Srivastava and Daoust function, given in (1.14), we arrive at the desired form presented as RHS of (2.6). This completes the proof of Theorem 2.2.

3. Special cases

In this section, we derive a number of corollaries of Theorem 2.1 and Theorem 2.2. Further, we establish connections of the corollaries with the other interesting and known results.

Corollary 3.1. *For $\rho = 1, \xi = 1$, and replacing ν by $\nu - 1$ with all the other conditions stated in Theorem 2.1, the following integral formula holds true:*

$$\int_{a_1}^{a_2} (t-a_1)^{\zeta-1} (a_2-t)^{\eta-1} [\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times E_{\omega,q,p}^{\nu,\gamma,\delta} \left[\frac{(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]dt$$

$$=\frac{(\mu^*+\lambda)^{(-\zeta)}(\mu^*+\mu)^{(-\eta)}}{(a_2-a_1)\Gamma(\nu)}B(\zeta,\eta)$$

$$\times F^{2+\lambda_1+\lambda_2+q:0;0}_{1+(\lambda_1+\lambda_2)+\omega+p:0;1}\left[\begin{matrix}[\Delta(\lambda_1;\zeta):1,2],[\Delta(\lambda_2;\eta):1,2],[1+\alpha+\beta:1,2],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta):1,2],[1+\alpha+\beta:1,1],\end{matrix}\right.$$

$$\begin{matrix}[\Delta(q;\gamma):1,1],[1+\alpha:1,1]:\_;\_;\\ [\Delta(p;\delta):1,1],[\Delta(\omega;\nu):1,1]:\_;[1+\alpha;1];\end{matrix}\frac{(\lambda_1{}^{\lambda_1})(\lambda_2{}^{\lambda_2})q^q}{p^p\omega^\omega(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(\mu^*+\lambda)^{\lambda_1}(\mu^*+\mu)^{\lambda_2}},$$

$$\left.\frac{-(\lambda_1{}^{2\lambda_1})(\lambda_2{}^{2\lambda_2})q^q}{p^p\omega^\omega(\mu^*+\lambda)^{2\lambda_1}(\mu^*+\mu)^{2\lambda_2}(\lambda_1+\lambda_2)^{2(\lambda_1+\lambda_2)}}\right]. \tag{3.1}$$

Corollary 3.2. *For $\nu=0,\ \omega=1,\ \rho=1,\ \xi=1,$ and with all the other conditions stated in Theorem 2.1, the following integral formula holds true:*

$$\int_{a_1}^{a_2}(t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times E^{\gamma,\delta}_{q,p}\left[\frac{(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\mu^*(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]dt$$

$$=\frac{(\mu^*+\lambda)^{(-\zeta)}(\mu^*+\mu)^{(-\eta)}}{(a_2-a_1)}B(\zeta,\eta)$$

$$\times F^{2+\lambda_1+\lambda_2+q:0;0}_{2+(\lambda_1+\lambda_2)+p:0;1}\left[\begin{matrix}[\Delta(\lambda_1;\zeta):1,2],[\Delta(\lambda_2;\eta):1,2],[1+\alpha+\beta:1,2],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta):1,2],[1:1,1],[\Delta(p;\delta):1,1],\end{matrix}\right.$$

$$\begin{matrix}[\Delta(q,\gamma):1,1],[1+\alpha:1,1]:\_;\_;\\ [1+\alpha+\beta:1,1]:\_;[1+\alpha;1];\end{matrix}\frac{(\lambda_1{}^{\lambda_1})(\lambda_2{}^{\lambda_2})q^q}{p^p(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(\mu^*+\lambda)^{\lambda_1}(\mu^*+\mu)^{\lambda_2}},$$

$$\left.\frac{-(\lambda_1{}^{2\lambda_1})(\lambda_2{}^{2\lambda_2})q^q}{p^p(\mu^*+\lambda)^{2\lambda_1}(\mu^*+\mu)^{2\lambda_2}(\lambda_1+\lambda_2)^{2(\lambda_1+\lambda_2)}}\right]. \tag{3.2}$$

Remark 3.3. *On taking* $\mu^* = 1,\ P_0^{(\alpha,\beta)}(z) = 1, \lambda_1 = \lambda_2 = \nu$, *Corollary* 3.2 *reduces into an interesting result derived by Pal (see* [18], *Theorem* 2.3, *p.4, Eq.*(2.4)*).*

Corollary 3.4. *For* $\rho = 1, \xi = 1$, *and replacing* ν *by* $\nu - 1$ *with all the other conditions stated in Theorem* 2.2, *the following integral formula holds true:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times E_{\omega,q,p}^{\nu,\gamma,\delta}\left[\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right]dudv$$

$$=\frac{(a_2-a_1)^{(2\zeta+\eta)}}{\Gamma(\nu)}B(\zeta,\eta)F_{1+(\lambda_1+\lambda_2)+\omega+p:0;1}^{2+\lambda_1+\lambda_2+q:0;0}\left[\begin{matrix}[\Delta(\lambda_1;\zeta):1,2],[\Delta(\lambda_2;\eta):1,2],\\ \\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta):1,2],\end{matrix}\right.$$

$$[1+\alpha+\beta:1,2],[\Delta(q,\gamma):1,1],[1+\alpha:1,1]:_-;_-;$$

$$[\Delta(\omega;\nu):1,1],[\Delta(p;\delta):1,1],[1+\alpha+\beta:1,1]:_-;[1+\alpha;1];$$

$$\frac{-({\lambda_1}^{\lambda_1})({\lambda_2}^{\lambda_2})q^q}{p^p\omega^\omega(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(a_2-a_1)^{-(2\lambda_1+\lambda_2)}},$$

$$\left.\frac{({\lambda_1}^{2\lambda_1})({\lambda_2}^{2\lambda_2})q^q}{p^p\omega^\omega(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{2(\lambda_1+\lambda_2)}}\right]. \tag{3.3}$$

Corollary 3.5. *For* $\nu = 0,\ \omega = 1,\ \rho = 1,\ \xi = 1$, *and with all the other conditions stated in Theorem* 2.2, *the following integral formula holds true:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times E_{q,p}^{\gamma,\delta}\left[\frac{(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right]dudv$$

$$=(a_2-a_1)^{(2\zeta+\eta)}B(\zeta,\eta)F^{2+\lambda_1+\lambda_2+q:0;0}_{2+(\lambda_1+\lambda_2)+p:0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta):1,2],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta):1,2],\end{array}\right.$$

$$[\Delta(\lambda_2;\eta):1,2],[1+\alpha+\beta:1,2],[\Delta(q,\gamma):1,1],$$

$$[1:1,1],[\Delta(p;\delta):1,1],[1+\alpha+\beta:1,1]$$

$$[1+\alpha:1,1]:\_;\_;\quad \frac{-(\lambda_1{}^{\lambda_1})(\lambda_2{}^{\lambda_2})q^q}{p^p(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(a_2-a_1)^{-(2\lambda_1+\lambda_2)}},$$

$$:\_;[1+\alpha;1];$$

$$\left.\frac{(\lambda_1{}^{2\lambda_1})(\lambda_2{}^{2\lambda_2})q^q}{p^p(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{2(\lambda_1+\lambda_2)}}\right]. \tag{3.4}$$

Corollary 3.6. *For $\xi=\rho=1$, and with all the other conditions stated in Theorem 2.2, the following integral formula holds true:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times J^{\nu,\gamma,\delta}_{\omega,q,p}\left[\frac{(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{(\lambda_1+\lambda_2)}}\right]dudv$$

$$=\frac{(a_2-a_1)^{(2\zeta+\eta)}}{\Gamma(\nu+1)}B(\zeta,\eta)F^{2+\lambda_1+\lambda_2+q:0;0}_{1+(\lambda_1+\lambda_2)+\omega+p:0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta):1,2],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta):1,2],\end{array}\right.$$

$$[\Delta(\lambda_2;\eta):1,2],[\Delta(q,\gamma):1,1],[1+\alpha+\beta:1,2],$$

$$[\Delta(p;\delta):1,1],[\Delta(\omega;\nu+1):1,1],[1+\alpha+\beta:1,1]$$

$$[1+\alpha:1,1]:\_;\_; \quad :\_;[1+\alpha;1]; \quad \frac{-(\lambda_1{}^{\lambda_1})(\lambda_2{}^{\lambda_2})q^q}{p^p\omega^\omega(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(a_2-a_1)^{-(2\lambda_1+\lambda_2)}},$$

$$\left.\frac{(\lambda_1{}^{2\lambda_1})(\lambda_2{}^{2\lambda_2})q^q}{p^p\omega^\omega(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{2(\lambda_1+\lambda_2)}}\right]. \tag{3.5}$$

Remark 3.7. *On setting* $a_1 = 0, a_2 = 1, \lambda_1 = \lambda_2 = 1,$ *and* $P_0^{(\alpha,\beta)}(z) = 1$, *in Corollary* 3.6*, we may produce a well-known result, investigated by Khan and Nisar (see* [13] *Theorem* 1*).*

4. Conclusion

In this paper, by applying the extensions of two classical lemmas, proposed earlier by Edward [5] and MacRobert [15], respectively, we have derived certain finite single and double integrals involving a product of a generalized Bessel-Maitland function and the classical Jacobi polynomial as the main result of the paper. Moreover, several corollaries are also obtained as the special cases of the main results derived in this paper. Further, the generalized Bessel-Maitland function may be reduced into several well-known classical functions, which have their potential applications in physics and other branches of applied science. Also, the popular Jacobi polynomial is directly connected with several other well-known classical polynomials. Thus, the obtained results are not only beneficial for the evaluation of several known forms of single and double finite integrals but also provide a general framework for the evaluation of numerous novel integrals.

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